

Name: _____

Exam 0 – 9/16/2022

Instructions

- You have 50 minutes to complete this exam.
- You may use your plebe-issue TI-36X Pro calculator.
- You may not use any other materials.
- **No collaboration allowed.** All work must be your own.
- **Show all your work.** To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.
- Keep this booklet intact.
- **Do not discuss the contents of this exam with any midshipmen until it is returned to you.**

Problem	Weight	Score
1a	1	
1b	1	
1c	1	
1d	1	
2a	1	
2b	1	
2c	1	
2d	1	
3a	1	
3b	1	
3c	1	
4a	1	
4b	1	
5a	1	
5b	1	
Total		/ 150

Problem 0. Copy and sign the honor statement below. This exam will not be graded without a signed honor statement.

The Naval Service I am a part of is bound by honor and integrity. I will not compromise our values by giving or receiving unauthorized help on this exam.

Problem 1. Consider the random variable X with the following pdf:

$$f_X(a) = \begin{cases} 0 & \text{if } a < -1, \\ \frac{3}{2}a^2 & \text{if } -1 \leq a < 1, \\ 0 & \text{if } a \geq 1. \end{cases}$$

a. What is the cdf of X ? Make sure to define the cdf $F_X(b)$ for all values of b between $-\infty$ and $+\infty$.

b. What is the expected value of X ?

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c. What is the minimum value that the random variable X can output? Briefly explain.

d. For the random variable X , which is more likely: a value near -1 , or a value near 0 ? Briefly explain.

Problem 2. Consider the random variable Y with the following cdf:

$$F_Y(a) = \begin{cases} 0 & \text{if } a < -3, \\ 0.2 & \text{if } -3 \leq a < 1, \\ 0.7 & \text{if } 1 \leq a < 7, \\ 1 & \text{if } a \geq 7. \end{cases}$$

a. What is the pmf of Y ?

b. What is the probability that $0 < Y \leq 4$?

c. What is the probability that $Y = 5$?

d. What is the variance of Y ?

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Problem 3. Simplex Pizza sells New York style and Sicilian style pizza by the slice. Let N represent the number of New York style slices in one order, and let T represent the total number of slices in one order. The joint pmf p_{NT} for N and T is:

p_{NT}		T		
		1	2	3
N	0	0.10	0.05	0.01
	1	0.25	0.10	0.02
	2	0	0.35	0.04
	3	0	0	0.08

a. What is the probability that an order contains a total of 2 slices?

b. Explain why $p_{NT}(2, 1) = p_{NT}(3, 1) = p_{NT}(3, 2) = 0$.

c. What is the probability that an order contains 2 New York style slices, given that the order contains a total of 2 slices?

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Problem 5. The Orange Company was having problems with its automated manufacturing cells yesterday: sometimes a tablet came out of a cell defective. 50% of the tablets were produced in cell 1, 30% in cell 2, and 20% in cell 3. 2% of the tablets produced in cell 1 came out defective, 3% in cell 2, and 5% in cell 3.

Suppose you select 1 tablet made yesterday at random. Let C be a random variable that represents the cell it was produced in (i.e., $C = 1, 2$ or 3). In addition, let D represent a random variable indicating whether the tablet came out defective (i.e., $D = 1$ if defective, 0 otherwise).

a. What is the probability that the randomly selected tablet came out defective, i.e. $\Pr\{D = 1\}$?

b. Are C and D independent? Give a numerical argument for why or why not.

Additional page for scratchwork or solutions